P P SAVANI UNIVERSITY

Second Semester of B. Tech. Examination

November 2022

SESH1080 Linear Algebra & Calculus

22.11.2022, Tuesday **Time: 01:00 p.m. To 03:30 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted compulsory.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Answer the Following: (Attempt any Five)

			CO	BTL
Q - 1	Check the set of all 2×2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with the standard matrix addition and multiplication is vector space or not.	[06]	1	3/5
Q - 2	Sets of vectors (1,1,2,1), (1,0,0,2), (4,6,8,6), (0,3,2,1) in R^4 are linearly dependent?	[06]	1	4
Q - 3	Determine the dimension and a basis for the solution space of the system $3x_1 + x_2 + x_3 + x_4 = 0$, $5x_1 - x_2 + x_3 - x_4 = 0$	[06]	1	2/4
Q - 4	Apply the Gram–Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$.	[06]	1	6
Q - 5	Find a QR –decomposition of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.	[06]	2	6
Q - 6	Let $F: R^3 \to R^2$ be the linear map defined by $F(x,y,z) = (3x + 2y - 4z, x - 5y + 3z)$. Find matrix of F in the following bases of R^3 and R^2. $S = \{w_1, w_2, w_3\} = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $S' = \{u_1, u_2\} = \{(1,3), (2,5)\}$.	[06]	2	4/6
Q - 7	Let $T: R^3 \to R^3$be the projection of a vector v into the xy-plane that is, $T(x,y,z) = (x,y,0)$. Find kernel and range.	[06]	2	3/5
Q - 8	Find the least square solution of the linear system $Ax = b$ given by $x_1 + x_2 = 7$, $-x_1 + x_2 = 0, -x_1 + 2x_2 = -7$ and find the orthogonal projection of b on the column space of A.	[06]	2	3/5

SECTION – II

Answer the Following:

Q - 1	Find all the local maxima, local minima, and saddle points of the function $f(x) = x^2 + xy + y^2 + 3x - 3y + 4$.	[05]	3	5
	OR			
Q - 1	Find all the local maxima, local minima, and saddle points of the function $f(x) = x^2 + xy + 3x + 2y + 5$.	[05]	3	5
Q - 2	Find $\frac{\partial w}{\partial v}$ when $u = 0, v = 0$ if $w = x^2 + \left(\frac{y}{x}\right), x = u - 2v + 1, y = 2u + v - 2$.	[05]	3	4
Q - 3	Trace the hypocycloid $x = a\cos^3 t, y = b\sin^3 t$.	[10]	4	6
	OR			
Q - 3	Trace the cardioid $r = a(1 - \cos\theta)$.	[10]	4	6

Q - 4	Write Legendre's duplication formula and evaluate $\Gamma\left(\frac{5}{2}\right)$.	**[05]**	**4**	**1/5**
Q - 5	**(1)** $\Gamma 1 =$ ____ **(2)** $\Gamma 0 =$ ____ **(3)** $\Gamma(1/2) =$ ____ **(4)** $\Gamma 2 =$ ____ **(5)** Symmetrical property of $B(m,n)$ is ____	**[05]**	**4**	**1/2**
Q - 6	Prove that $n\,B(m+1,n) = m\,B(m,n+1)$	**[05]**	**4**	**5**

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CO : Course Outcome Number BTL : Blooms Taxonomy Level

Level of Bloom's Revised Taxonomy in Assessment

1: Remember	2: Understand	3: Apply
4: Analyze	5: Evaluate	6: Create